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A New Look at the Cross-Impact Matrix and its Application in Futures Studies

Kenneth Chao
Kainan University
Taiwan

Abstract

In this paper, the author presents a new approach to cross-impact matrix analysis which combines the level of anticipated future impacts and their probability of occurrence, so that both are factors in the analysis. Its combined result will serve as a basis for evaluating the future development.

Keywords: cross-impact matrix, conditional probability, initial probability, likely impact, expected impact

Introduction

Theodore Gordon and Olaf Helmer first developed the cross impact-matrix method of analysis in 1965 in a game called "Futures" for Kaiser Aluminum and Chemical Company on the occasion of their 50th anniversary. It has received great attention and has been used as a major method for Futures Studies. Many articles have been written on it. Many researchers have revised the method to be more applicable (Duperrin & Godet, 1975; Fontela, 1976; Helmer, 1977; Enzer & Alter, 1978; Sarin, 1978; Novak & Lorant, 1978; Wissema & Benes, 1980; Hanson & Ramani, 1988). The success of cross-impact method lies in its systematic analysis of interactions among possible future developments.

A cross-impact matrix is a $n \times n$ matrix $[\, a_{ij} \,]$.

Each cell or entry of the matrix, a_{ij}, represents the impact on (or conditional probability of) event i given the occurrence of event j. For a 3-event cross impact matrix, we have

$$\begin{array}{c} \\ Event\ 1 \\ Event\ 2 \\ Event\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} Event\ 1 & Event\ 2 & Event\ 3 \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \end{array}$$

For example, a_{12} represents the impact to (or conditional probability of) event 1 given the occurrence of event 2.

Some users of the cross-impact matrix prefer to use a_{ij} to represent the impact of (or conditional probability of) event *j* given the occurrence of event *i*. That is, reverse the role of row and column, as T.J. Gordon (1965) originally designed in his cross impact matrix. In essence, these two conventions are simply the transposition of each other. The meaning of the entries of the cross-impact matrix has been defined differently as well. There have been 3 major categories in their meanings when the cross-impact matrix was used.

Type 1. Qualitative description of the trend-scenario.
(e.g. Ratcliffe, 2001; Ambrose, 2002)
Type 2. Trend value, i.e. magnitude of impact, for example, on a scale of 1 to 5.
(e.g. Gordon & Hayward, 1969; Twiss, 1992; Chen Kuo-Hua, 1999; Weerakkoday & Tremblay, 2003)
Type 3. The conditional probability.
(e.g. Dalkey, 1972; Enzer, 1972; Gordon, 1994)

A Few Observations

It seems that all three approaches to the cross-impact matrix work well in their own domains, but function independently of each other.

The scenario approach Type 1 describes impacts but does not contain quantitative information for a sense of measurement. Its value lies in helping to identify causality chains for later scenario construction, that is, "if i happens, then j follows..." The trend value cross-impact matrix Type 2 approach quantifies impacts of the events on each other so that further analysis of the matrix is possible and changes in initial probability assumptions can be made on the basis of the net effect of the interactions. Type 3 cross-impact matrices analyses are based on assumed quantitative conditional probabilities which appear in the cells of the matrix, and the solution of these matrices leads to a re-estimate of assumed initial probabilities for all of the events depicted in the matrix.

The writer sought to combine both the impact and its probability in this currently proposed approach by having both the conditional probabilities of impact and the overall probability of each event appear in the matrix cells.

The Proposed Approach

In this article, the writer would like to suggest an alternative approach to the cross-impact matrix, allowing for some further applications.

1. Both Type 2 and Type 3 cross-impact matrices will be used. For Type 2, we can use a scale of, say, -5 to +5 to indicate the level and direction of impact. Use -5 to mean the most negative impact and +5 as the most positive impact. Of course, other scales can be used to suit the individual research.

Type 3 will be defined in a slightly different manner. Instead of "if the column events were to occur, then what is the probability that the row events will occur?" it will now have the meaning "if the column events were to occur, then what is the probability that the magnitude of the impact to the row events defined in the associated impact level cross-impact matrix will occur?"

2. In almost all uses of the cross-impact matrix, the diagonal was left blank, as the following example from an article "Cross-Impact Method" by T. J. Gordon (1994), shows:

Table 1. *Corss-impact matrix*

If This Event Occurs	**The Probability of these Events Becomes:** Event 1	Event 2	Event 3	Event 4
Event 1 (0.25)		0.50	0.85	0.40
Event 2 (0.40)	0.60		0.60	0.55
Event 3 (0.75)	0.15	0.50		0.60
Event 4 (0.50)	0.25	0.70	0.55	

Note: Values in parentheses are initial probabilities.
Values in the matrix are conditional probabilities.

In the new approach presented here, the diagonal entries will not be blank. Instead, the conditional probabilities or impact to that event will be positioned there. For the conditional probability cross-impact matrix, the diagonal entries will be filled with 1's. The interpretation is that if an event occurs, then the probability of that particular event's occurrence is 1 (100%). As for the level of impact cross-impact matrix, the diagonal element will be the initial impact to each event in the absence of the cross impacts. The reason is that if that event occurs, there should be an impact. In other words, it should mean something (positive, negative or otherwise) to us.

3. The initial probabilities of occurrence for the events will be formed as an initial probability vector ($n \times 1$ column vector) as follows.

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$

where p_i is the initial probability for event i for $i = 1, 2, ..., n$.

We will describe the process of this new approach now. Let A be a n x n cross-impact matrix with n possible events or trends. The entries a_{ij} ($1 \le i \le n$; $1 \le j \le n$) are defined as follows.

Table 2. *Revised cross-impact matrix*

How does this ⟹ Driving event
Affect this ⇓

$$A = \begin{array}{cc} & \\ \text{Driven} & \\ \text{event} & \end{array} \begin{array}{c} \textit{Event } 1 \\ \textit{Event } 2 \\ \vdots \\ \textit{Event } i \\ \vdots \\ \textit{Event } n \end{array} \overset{\begin{array}{cccccc} \textit{Event } 1 & \textit{Event } 2 & \cdots & \textit{Event } j & \cdots & \textit{Event } n \end{array}}{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{bmatrix}}$$

We notice that the driving events are the column events, the driven events are the row events. That is, the matrix represents "if the column events were to occur, what is the magnitude of the impact to the row events?"

We will define $B = [\, b_{ij} \,]$ similar to $A = [\, a_{ij} \,]$. Matrix B now represents "if the column events were to occur, what is the conditional probability of impact of the row events?"

Next we define an initial probability vector as an n-dimensional vector.

$$P = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$

where p_i = the initial probability of the occurrence of event i for $i = 1, 2, ..., n$.

If we denote E_i to be event i for $i = 1, 2, ..., n$ then $p_i = P(E_i)$.

We will begin by creating a new matrix $X = [\, x_{ij} \,]$ where $x_{ij} = a_{ij}b_{ij}$ which represents the likely impact of event i given the occurrence of event j. This number x_{ij} can serve as a basis for evaluating the risk in comparison to alternatives.

For example, if an event has an impact of 4 and its conditional probability of impact is 50%, then its likely impact is 4x0.5 = 2. Now consider an event that has an impact of 4 but a conditional probability of impact 90%. The likely impact would be 4x0.9 = 3.6. This higher value indicates that the second case is a better scenario than the first case. Matrix X is then formed by multiplying each element of A by the corresponding element of B. X can be thought of as a matrix representing the likely cross-impact.

Next, we perform a matrix multiplication XP.

Let's see what will happen if we perform a multiplication of XP. Let $S = XP$, then

$$XP = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ x_{n1} & \cdots & \cdots & x_{nn} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} = S$$

$$\begin{aligned} s_i &= \sum_{j=1}^{n} x_{ij} p_j \\ &= \sum_{j=1}^{n} (a_{ij} b_{ij}) p_j \\ &= \sum_{j=1}^{n} (a_{ij} b_{ij}) P(E_j) \end{aligned}$$

This is an expected impact of event i given the probability of occurrence of each and all events. This vector contains n entries. It represents the expected impact of each event.

For example, for a 3 x 3 case, we have

$$\begin{matrix} & E_1 & E_2 & E_3 \\ \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} & \left[\begin{matrix} a_{11} \\ a_{21}b_{21} \\ a_{31}b_{31} \end{matrix}\right. & \begin{matrix} a_{12}b_{12} \\ a_{22} \\ a_{32}b_{32} \end{matrix} & \left.\begin{matrix} a_{13}b_{13} \\ a_{23}b_{23} \\ a_{33} \end{matrix}\right] \end{matrix} \begin{bmatrix} P(E_1) \\ P(E_2) \\ P(E_3) \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}P(E_1) + (a_{12}b_{12})P(E_2) + (a_{13}b_{13})P(E_3) \\ (a_{21}b_{21})P(E_1) + a_{22}P(E_2) + (a_{23}b_{23})P(E_3) \\ (a_{31}b_{31})P(E_1) + (a_{32}b_{32})P(E_2) + a_{33}P(E_3) \end{bmatrix} = S$$

S is then the vector representing the expected impact of each and every event.

Let's take an example to illustrate the procedure. A family has the following impact level cross-impact matrix. Its elements represent the impact to the family welfare.

Table 3. *Revised cross-impact matrix*

How does this ⇨ / Affect this ⇩	Unhealthy Marriage	Sound Finance	Outstanding Children
Unhealthy Marriage	−5	−3	2
Sound Finance	2	3	4
Outstanding Children	−2	2	5

Here we use the scale of -5 to +5 to indicate the level and direction of impact.

In this example, the a_{11} position has the value of -5 which means that "Unhealthy Marriage" has the most negative impact to family welfare. The value of +5 in the a_{33} position means that "Outstanding Children" has the most positive impact to family welfare. The value of -3 in the a_{12} position means that "Sound Finance" contributes to "Unhealthy Marriage" in a negative manner with an impact value of -3 to family welfare.

It has the following associated conditional probability cross-impact matrix.

Table 4. *Conditional probability cross-impact matrix*

Then the probability of impacting these events would be	If this event were to occur: Unhealthy Marriage (0.7)	Sound Finance (0.5)	Outstanding Children (0.4)
Unhealthy Marriage	1	0.8	0.5
Sound Finance	0.7	1	0.4
Outstanding Children	0.6	0.5	1

Values in parentheses are initial probabilities.

Values in the matrix are conditional probabilities.

The initial probability 0.7 for "Unhealthy Marriage" means that it has a 70% chance of occurring without consideration of the cross-impacts. The conditional probability of 0.8 in the b_{12} position means that the probability that "Sound Finances" will impact "Unhealthy Marriage" is 80%. The justification is that there is no guarantee that "Sound Finances" will definitely impact "Unhealthy Marriage." The probability of materialization of the impact of -3 in the previous impact level cross-impact matrix is 80%.

So we have the following cross-impact matrices.

$$A = \begin{bmatrix} -5 & -3 & 2 \\ 2 & 3 & 4 \\ -2 & 2 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0.8 & 0.5 \\ 0.7 & 1 & 0.4 \\ 0.6 & 0.5 & 1 \end{bmatrix}$$

and an initial probability vector $P = \begin{bmatrix} 0.7 \\ 0.5 \\ 0.4 \end{bmatrix}$

then,

$$X = \begin{bmatrix} -5 & -2.4 & 1 \\ 1.4 & 3 & 1.6 \\ -1.2 & 1 & 5 \end{bmatrix}$$

$$S = XP = \begin{bmatrix} -4.3 \\ 3.12 \\ 1.66 \end{bmatrix}$$

$$Sum \quad \overline{0.48}$$

The vector *S* shows that the expected impact of unhealthy marriage to the family welfare is -4.3. The expected impact of sound finance to the family welfare is 3.12. The expected impact of outstanding children to the family welfare is 1.66. Moreover, the sum of the entries of *S*, 0.48, represents the net expected impact from the cross-impact matrices. It can serve as a basis for evaluation of future development.

For example, suppose another set of trend-value and conditional probability cross-impact matrices produces a different net expected impact, then a comparison can be made and further analysis can be done to the two different trend-scenarios.

Conclusions and Recommendations

The purpose of this paper is to explore another approach of the cross-impact matrix technique. It is by no means complete. The author wishes the colleagues interested in the methodology of Futures Studies will offer valuable comment and revision to this approach so that it may be useful to our Futures Studies.

The overall differences between this new approach and the previous cross-impact matrix techniques are as follows.

Table 5. *Comparison of previous and new technique*

Previously Techniques	New Approach
The probability oriented cross-impact matrix and the impact oriented cross-impact matrix were used independently.	The approach combines both the impact level and the probability.

We list below follow-up questions that may be raised for the new approach.

1. Can we perform sensitivity analyses of the initial probability? That is, how changes in the initial probability affect or alter the expected impact of each event? This sensitivity analysis was suggested by S. Enzer (1972) in his paper

"Cross-Impact Techniques in Technology Assessment" to produce probability change after consideration of interaction among events.

2. Can the approach be incorporated in other models (e.g. econometric or system dynamic) to introduce the consequences of external events on otherwise extrapolative approaches?

Correspondence

Kenneth Chao
Department of Accounting Information
Kainan University
Taiwan
Tel: +886-3-341-2500 # 6113
Mobile phone: 0955-326-186
Fax: +886-3-341-2196
33857 No.1, Kainan Rd., Luzhu, Taoyuan County, Taiwan
Email: kenkchao@mail.knu.edu.tw

References

Chen, Kuo-Hua. (1999). The prediction and research method for Futures Studies. *Futures Studies in Sociology.* pp. 119-157. Taipei, Taiwan: Hwatai.

Dalkey, Norman. (1972). An elementary cross impact model. *Technological Forecasting and Social Change, 3* (3), 341-351.

Enzer, Selwyn. (1972). Cross-impact techniques in technology assessment. *Futures*, *4*(1), 30-51. Torino, Italy: Elsevier.

Gordon, Theodore.(1994).Cross-impact method. *Future Research Methodology.* Washington DC: AC/UNV Millennium Project.

Gordon, Theodore, & H. Hayward (1968). Initial experiments with the cross impact Matrix Method of Forecasting. *Futures*, *1* (2), 100-116. Torino, Italy: Elsevier.

Helmer, Olaf. (1972). Cross-impact gaming. *Futures*, *4* (2), 149-167.

Ratcliffe, John. (2001). Futures Studies: A background paper to promote the study of futures for the real estate industry. *The Cutting Edge 2001.* Oxford, 6th September 2001. London: South Bank University.

Weerakkody, Niranjala & Wilfred Tremblay. (2003). Digital TV in Australia and the U.S.A.: A cross-impact analysis of the adoption and diffusion of digital TV in Australia and the United States. ANZCA03 Conference, Brisbane, July 2003.